THE FORCE PRODUCED AND OBSERVED BY AN ASYMMETRIC CAPACITOR THRUSTER AS MEASURED IN HORIZONTAL AND VERTICAL CONFIGURATIONS

Lisa M. Kotowski  
Department of Physics and Astronomy  
University of Hawai‘i at Mānoa  
Honolulu, HI 96822

ABSTRACT

Throughout the spring 2013 semester I continued to work on the frictional model for a rotating lifter system. The previous frictional model assumed friction to be a constant, which only showed to be valid for sufficiently small values of $\theta$. Now it has been determined that friction is proportional to the velocity, which is valid through all angular displacements. Allowing the lifters to move through air, a non-viscous liquid, $\beta = 2$, allowing us to determine that $k = (0.02708516 \pm 0.0006465) \frac{kg}{m}$. This value of $k$ was then used to evaluate the thrust force of a two-armed rotating lifter system in both the horizontal and vertical orientations. In the horizontal orientation of the rotating lifter system, the magnitude of the thrust force was determined to be $F_L = (163.8856 \pm 0.0072) mN$. Moreover, in the vertical orientation of the rotating lifter system, the magnitude of the thrust force was determined to be $F_L = (184.8056 \pm 0.0939) mN$. The magnitude of the thrust force calculated for the vertical configuration of the system is consistent with the magnitude of the thrust force calculated for the horizontal configuration of the rotating lifter system.

INTRODUCTION

An asymmetrical capacitor thruster (ACT) consists of a small electrode and a large electrode separated by a small air gap and connected by a high voltage as shown below in Figure [1]. When the high voltage is applied, the air surrounding the electrodes ionizes, the charged air molecules then begin to flow from the small electrode to the large electrode in a downwards fashion. This flow of ionized molecules is known as the Biefield-Brown effect, and yields a force sufficient enough to lift the ACT.

![Figure 1: A diagram depicting the basic structure of an ACT, and the direction of flow for the ionized air molecules.](image-url)
The remarkable thing about an ACT is that it acts like a motor with no moving parts, making it an interesting topic of research for propulsion. Together with Team Ion Flow and James Bynes on a rotating lifter system has been created in order to study the characteristic motion and properties of an ACT. Over the course of this semester, our build team (consisting of Team Ion Flow and James Bynes) constructed an enclosure to house the experimental setup; meanwhile, I developed a frictional model and analyzed the characteristic motion of the rotating ACT system.

Figure 2: A Solidworks rendering of the rotating ACT system. It can be configured either: vertically (pictured) such that the axis of rotation is perpendicular to the direction of gravity or horizontal such that the axis of rotation is parallel to the direction of gravity.

**THEORY/EXPERIMENTAL METHODS**

**Unpowered Frictional Analysis**

We begin the frictional analysis through all angular displacements by using an unpowered ionocraft system, and allowing it to rotate until it reaches rest. The Torque for the system will be analyzed in order to determine the frictional force observed by the ionocrafts.

\[
\Sigma \tau = I \ddot{\theta} = I \ddot{\alpha}
\]

\[
\tau = \Sigma (d \times F) = I \ddot{\alpha}
\]

In this scenario the lifter system is unpowered, and therefore only the frictional force is present; which, in the horizontal equation shown in Equation [1] is modeled as \( k v^\beta \).

\[
-fr = I \alpha
\]

\[
\therefore (-kv^\beta)r = (-k(\tau^\beta)r = I \alpha = I \frac{d\omega}{dt}
\]

\[
\Rightarrow -kr^{\beta+1}\omega^\beta = \frac{d\omega}{dt}
\]
Here equation [3] describes the generalized form of the unpowered system moving through a medium. Analysis for movement through a viscous fluid ($\beta = 1$) and for movement through a non-viscous fluid ($\beta = 2$) were performed, and best fits were made to the data in order to determine the value for the drag coefficient $k$.

**Unpowered Movement Through a Viscous Fluid**

Supposing that the ACT were moving through a viscous fluid we would assume $\beta = 1$, therefore the differential equation becomes:

\[
-\frac{k r^2}{I} dt = \frac{d\omega}{\omega}
\]

\[
\int_{0}^{t} -\frac{k r^2}{I} dt' = \int_{\omega_{0}}^{\omega} \frac{d\omega'}{\omega'}
\]

\[
-\frac{k r^2}{I} t' \bigg|_{0}^{t} = \left[ \ln(\omega') \right]_{\omega_{0}}^{\omega}
\]

\[
-\frac{k r^2}{I} t = \ln \omega - \ln \omega_{0} = \ln\left(\frac{\omega}{\omega_{0}}\right)
\]

\[
\exp\left(-\frac{k r^2}{I} t\right) = \frac{\omega}{\omega_{0}}
\]

\[
\therefore \omega = \omega_{0} \exp\left(-\frac{k r^2}{I} t\right) = \frac{d\theta}{dt}
\]

From this we are able to determine the angular displacement by integrating equation [4] with respect to time.

\[
\omega = \frac{d\theta}{dt} = \omega_{0} \exp\left(-\frac{k r^2}{I} t\right)
\]

\[
d\theta = \left(\omega_{0} \exp\left(-\frac{k r^2}{I} t\right)\right)dt
\]

\[
\int_{\theta_{0}}^{\theta} d\theta' = \int_{0}^{t} \left(\omega_{0} \exp\left(-\frac{k r^2}{I} t'\right)\right)dt'
\]

\[
\theta - \theta_{0} = \omega_{0} \left(\frac{I}{k r^2} \exp\left(-\frac{k r^2}{I} t\right) + \frac{I}{k r^2}\right)
\]

\[
\theta = \frac{\omega_{0} I}{k r^2} \left(1 - \exp\left(-\frac{k r^2}{I} t\right)\right) + \theta_{0}
\]

89
Unpowered Movement through a Non-Viscous Fluid

If the lifter is moving through a non-viscous fluid then we assume $\beta = 2$, therefore Equation [3] becomes:

$$\frac{-kr^3}{I} \frac{d\omega}{\omega^2}$$

$$\int_0^t dt' = \frac{-I}{kr^3} \int_0^{\omega} \frac{d\omega'}{\omega'^2}$$

$$t = \frac{I}{kr^3} \left( \frac{1}{\omega} - \frac{1}{\omega_o} \right)$$

$$\frac{1}{\omega_o} + \frac{kr^3}{I} t = \frac{1}{\omega} = \frac{I + \omega_o kr^3 t}{I \omega_o}$$

$$\frac{d\theta}{dt} = \omega = \frac{I \omega_o}{I + \omega_o kr^3 t}$$

Equation [6]

From this we are able to determine the angular displacement by integrating equation [6] with respect to time.

$$\int_{\theta_o}^{\theta} d\theta' = \int_0^t \frac{I \omega_o}{I + \omega_o kr^3 t'} dt'$$

$$\theta = \frac{I}{kr^3} \ln \left[ \frac{I + \omega_o kr^3 t}{I} \right] + \theta_o$$

Equation [7]

Powered Frictional Analysis

In Section 2.1 we determined the value of the frictional drag coefficient $k$ for the rotating ionocraft system, which allows us to analyze the data collected while the system is powered in order to determine the thrust force produced by each ACT. Again we begin by analyzing the vector torque of the rotating ACT system:

$$\tau = \Sigma (\vec{d} \times \vec{F}) = I \vec{\alpha}$$

$$NF - kv^2 = I \alpha$$

Equation [8]

where $N$ is the number of ACT's attached to the rotating ACT system. Since the ACT's are moving through air, a non-viscous fluid; theoretically, the frictional model for $\beta = 2$ has been shown to better describe the friction of the system.
Movement Through a Non-Viscous Fluid

Using the drag coefficient for movement through a viscous fluid ($\beta = 2$) for a 2-armed ACT system, Equation[8] then becomes:

$$(2F - kr\omega^2)r = 2Fr - kr^3\omega^2 = I\alpha = \int_0^t \frac{d\omega}{dt}$$

$$\int_0^t dt' = \int_{\omega_o}^{\omega} \frac{d\omega'}{F - r^2k\omega'^2}$$

$$t = \frac{I}{r^3k} \int_{\omega_o}^{\omega} \frac{d\omega'}{\frac{F}{r^2k} - \omega'^2}$$

$$\frac{r^3kt}{I} = \frac{1}{2\sqrt{\frac{2F}{kr}}} \left[ \ln \left( \frac{\sqrt{\frac{2F}{kr}} + \omega}{\sqrt{\frac{2F}{kr}} - \omega} \right) \right]_{\omega_o}^{\omega}$$

Therefore, assuming the system begins at rest (i.e. $\omega_o = 0$), we find that Equation [9] becomes:

$$\frac{kr^3t}{I} \frac{2F}{kr^2} = \ln \left( \frac{\sqrt{\frac{2F}{kr}} + \omega}{\sqrt{\frac{2F}{kr}} - \omega} \right)$$

$$e^{\frac{kr^3t}{I} \frac{2F}{kr^2}} = \frac{\sqrt{\frac{2F}{kr}} + \omega}{\sqrt{\frac{2F}{kr}} - \omega}$$

$$\int_{\theta_o}^{\theta} d\theta' = \sqrt{\frac{2F}{kr^2}} \int_{q'}^{t} \left[ \frac{\exp\left(\frac{2r^2}{I} \sqrt{2Fkt'}\right) - 1}{\exp\left(\frac{2r^2}{I} \sqrt{2Fkt'}\right) + 1} \right] dt'$$

$$\theta = \sqrt{\frac{2F}{kr^2}} \left[ -t + \frac{I}{r^2\sqrt{2Fk}} \ln \left( \frac{1 + \exp\left(\frac{2r^2}{I} \sqrt{2Fkt}\right)}{2} \right) \right]$$

Equation [11] can then be fit to data collected when the system is powered in order to determine the value of the thrust force.

DATA ANALYSIS

Data was collected multiple times for the powered and unpowered system in horizontal and vertical configurations. Using a Solidworks rendering of the system, it was determined that the moment of Inertia ($I$) is 0.00155kg$m^2$; moreover, the radius of the rotating ionocraft arm was measured to be 0.06m. Using these values as constants, the value of the drag coefficient ($k$) can be determined using a best-fit line.
From the fit data collected from the best-fit line the drag coefficient for the unpowered two-arm system was determined to be $(0.03208144 \pm 0.00035486) kg s^{-1}$ if the ACT's are moving through a viscous fluid and $(0.02708516 \pm 0.000647) kg s^{-2}$ if the ACT's are moving through a non-viscous fluid. The results were confirmed by comparing the viscous and non-viscous fluid fits, it was determined that the drag coefficient when the rotating ACT system is subjected to air follows the movement of an ACT through a non-viscous fluid.

Data Analysis: Determining the Thrust Force

Data for the powered rotating ACT system was collected for both the horizontal and vertical configurations of the two-armed configuration of the ACT system. Due to the even number of arms of the experimental setup, the vector torque of the vertical two-armed system is the same as that of the horizontal two-armed system. This allows us to analyze the data by fitting Equation [11] data collected for each configuration. Using the value of $k$ determined from the unpowered rotating ACT system, we are able to fit Equation [11] to the collected data in order to determine the magnitude of the thrust force.

Figure 3: A plot with a best-fit line depicting how friction affects the one-armed horizontal lifter system through all values of $\theta$. a) the best fit for movement through a viscous fluid ($\beta = 1$) and b) the best fit for movement through a non-viscous fluid ($\beta = 2$).

Figure 4: a) The best-fit line for the powered horizontal configuration of the two-arm ACT system. b) The best-fit line for the powered vertical configuration of the two-arm ACT system.
From the best fit lines in Figure [4] the thrust force was determined to be \((163.8856 \pm 0.0072) mN\) for the powered horizontal two-arm system, and \((184.8056 \pm 0.0939) mN\) for the powered vertical two-arm system. The thrust force measured for the vertical configuration is consistent with the thrust force measured for the horizontal configuration.

**FUTURE RESEARCH**

Although this project has determined the thrust force produced by each traditionally-shaped ACT, in the future it would be interesting to see how the geometry of ACT's affects the thrust force. This can be investigated in three ways: 1) by increasing the surface area of a traditional lifter, 2) changing the geometry of the ACT, and 3) stacking multiple ACT's together to combine their thrust forces. I think these could increase the thrust force produced by one ACT and become more efficient for using a thrust mechanism.

**CONCLUSIONS**

The angular nature of the rotating ACT system was examined using the vector torque of the system when unpowered and when powered. When the ACT system was unpowered, we were investigating the value of the frictional drag coefficient. After fitting the data and comparing the drag coefficient for movement through a viscous fluid and movement through a non-viscous fluid, it was determined that the analysis for the non-viscous fluid was more ideal. This confirmed the theoretical frictional model because the ACT's move through air, which is a non-viscous fluid. The drag coefficient for the unpowered two-arm system was determined to be \((0.03208144 \pm 0.00035486) kg \; s^{-1}\) if the ACT's are moving through a viscous fluid and \((0.02708516 \pm 0.000647) kg \; s^{-2}\) if the ACT's are moving through a non-viscous fluid. By visually comparing the viscous and non-viscous fluid fits, it was determined that the drag coefficient when the rotating ACT system is subjected to air follows the movement of an ACT through a non-viscous fluid.

The thrust force was determined to be \((163.8856 \pm 0.0072) mN\) for the powered horizontal two-arm system, and \((184.8056 \pm 0.0939) mN\) for the powered vertical two-arm system. The thrust force measured for the vertical configuration is consistent with the thrust force measured for the horizontal configuration.

**ACKNOWLEDGEMENTS**

The results yielded by this experiment would not be possible without the effort from Team Ion Flow in designing and constructing the experimental Apparatus. I would especially like to thank Holm Smidt and Liem Nguyen for their hard work and long hours helping to fine tune the experimental setup. Moreover, this project wouldn't have been possible without the leadership and devotion of James Bynes. Finally, I would like to thank my mentor Herve Collin, who helped keep us on track and determine the best ways to create the experiment. Thank you all for your hard work this past year, it has been a pleasure to work with all of you!
REFERENCES


