MICROLENSING: A NONPARAMETRIC STATISTICAL MODEL OF MILKY WAY HALO MASSES

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ABSTRACT

The following paper will apply nonparametric statistical modeling to the technique of gravitational microlensing to better understand the nature and scope of non-luminous matter (dark matter) in the Milky Way and elsewhere. Microlensing data gathered towards the galactic halo (viewed against the background of the Magellanic Clouds) will be the primary source of data for input into the model. The rotational curve of our galaxy indicates large amounts of this dark matter outside of the suns distance from the center (galactic bulge) as defined by Newtonian Mechanics. Microlensing events may represent the only observations of an otherwise unobservable component of the Galaxy and therefore their interpretations in terms of mass is of great interest. Understanding and accounting for the distribution of mass in our galaxy (luminous and non-luminous) is essential in constructing and developing expansion theories. These understandings could help on larger scales in Cosmology studies and universal expansion theories.

INTRODUCTION

Since the introduction of gravitational microlensing by Paczynski (1986, 1991) there have been principally two fields of observing, the galactic halo (viewed against the background of the Magellanic Clouds) and thick disc (viewed against the galactic bulge). Collaborations of scientist have been, and continue to, routinely image fields of large numbers of background stars for the variability signature of microlensing. Despite the large number of stars monitored (typically on the order of 10^6), the rarity of events has limited the number of detections to a few tens. Interpretation in terms of mass is complicated by the dependence of observable microlensing parameters on multiple unknown values, including mass. Important among these are the distance to the encounter, its velocity, and the distances to the lens and light source (background stars). To arrive at values for the lens mass requires estimates of an inherently statistical nature. This is usually accomplished by using mean values for the unknown quantities derived from kinematical models of the halo and thick disc. Since the individual masses of microlenses are not of as great an interest as their overall mass distribution, it is possible to directly infer the mass distribution from the microlensing observables based upon an explicit model of lens kinematics. Such statistical approaches would have the advantage of estimating the mass function directly from the distributions of observables and assumed kinematical model, without the possibly distorted effects of average value estimates of velocities and distances. It has been our goal to derive such a model and apply it to all detected halo microlenses to estimate the dark halo mass distribution *and the sensitivity of such an estimate to the presumed form of the kinematical model.*

METHODS

To develop the model we first look at the independent variables involved in an observed microlens:

M	Lens mass
D	Lens distance (from observer)
D_S	Source distance (from observer)
V	Encounter velocity on plane of sky
b	Encounter closest approach on plane of sky

The Einstein radius is given below:

$$R_E = \frac{2}{c} \cdot \sqrt{\frac{GMD(D_s - D)}{D_s}}$$

Here c is the speed of light and G the gravitational constant. The observed amplitude yields the factor β and the timescale yields the event frequency ω.

$$\beta = \frac{b}{R_E} \qquad \omega = \frac{V}{R_E}$$

β and ω contain all obtainable information and are functions of 5 independent variables. No unique solution exists for the lens mass due to the lens distance, velocity, and distance to closest approach being unobservable. For this we utilize statistical distributions for these variables and construct a statistical model that allows the distribution of the lens mass to be extracted from the distributions of observed β and ω for lenses in the galactic halo. Nonparametric statistical modeling provides the best solution for such a problem, as previously applied in Astronomy (*e.g.* Heacox 1998).

Limitations of the model exist from the a priori distribution of distances of closest approach, b; therefore the observed lens amplitude and β contain no useful information on the lens mass. The statistical model is consequently constructed entirely from the observed microlensing frequency ω. A generic form of the model is given as:

$$f_\omega(\omega) = \int f_M(M)\, K(\omega, M)\, dM$$

The observed probability density function (pdf) of lens frequencies $f_\omega(\omega)$ is given in terms of the pdf of lens masses $f_M(M)$ and the integral kernel K (the conditional probability of observing a lens frequency ω given a lens mass M. Given a kinematical model for the halo that includes the joint distribution of velocities and galactocentric distances the required kernel can be computed. Even though the credibility of the mass function estimate decreases with the need for a kinematical model the use of such assumptions allows easy estimation of the sensitivity of the results.

In nonparametric statistical modeling applied to halo microlenses viewed against the Magellanic Clouds the parameter D_S is not a stochastic variable and can be presumed known. The derivation of the simplified model is given as:

$$f_\omega(\omega) = \int^{\infty}\int^{\infty} f_{\omega,D,M}(\omega, D, M)\,dD\,dM$$

Making substitution leads to:

$$f_\omega(\omega) = \int^{\infty}\int^{\infty} f_{V,D,M}(V(\omega, D, M), D, M)\cdot\left|\frac{\delta V}{\delta\omega}\right| dD\,dM$$

$$f_\omega(\omega) = \int^{\infty}\int^{\infty} f_{V,D,M}\left[\omega\cdot R_E(D, M), D, M\right]\cdot R_E(D, M)\,dD\,dM$$

We now make a necessary assumption that the lens mass M is independent of the velocity and distance D so that $f_{V,D,M}(V, D, M) = f_{V,D}(V, D)\cdot f_M(M)$ and the last equation can be written in the form of the generic model with the kernel:

$$K(\omega, M) = \int^{\infty} f_{V,D}\left[\omega\cdot R_E(D, M), D\right]\cdot R_E(D, M)\,dD$$

The application of the statistical model with this kernel to a set of microlens observations is a kinematical model, consisting of (1) an assumed joint distribution of velocities and solar centric distances $f_{V,D}$; and (2) the assumption that lens masses are independent of velocities and distances.

Solving the model for the desired f_M, given the observed f_ω and the above assumptions, is best done with a form of the Richardson-Lucy iterative algorithm (Lucy 1974).

$$\left(f_M\right)^{<n+1>}(M) = \left[\left[\left(f_M\right)^{<n>}\right](M)\right]\cdot\int^{\infty}\frac{K(\omega, M)\,f_\omega(\omega)}{\int^{\infty} K(\omega, M')\left(f_M\right)^{<n>}(M')\,dM'}\,d\omega$$

where $(f_M)^n(M)$ is the n^{th} iterate of the procedure, and $f_\omega(\omega)$ is the observed lens frequency distribution. Starting with an initial assumed mass distribution one iterates until sensible convergence.

DISCUSSION

Application of the model to galactic halo lenses viewed against the background of the Large Magellanic Cloud is reasonably good place to start due to the distance to the source being relatively well known, and the luminous halo kinematics are partially understood bettering the needed derived kinematical model. The observed velocities in the luminous halo appear to arise from a non-rotating spheroidal component dominated by large random velocities of approximately Gaussian distributions (Binney & Merrifield 1998). This implies that the relative encounter velocities of the halo microlenses are dominated by: (1) the Sun's galactocentric motion, (2) the large proper motion of the Magellanic Clouds (Jones *et al.* 1994), and (3) the random galactocentric velocities of halo objects. It is probable that the velocity dispersion is probably a decreasing function of galactocentric distance, but since most lensing events take place at small distances there is no good evidence for such a dependence and we assume that halo velocities are independent of distance.

Computing the integral kernel brings forth noteworthy points. The frequency distribution to be observed for a set of lenses of all the same mass is approximately of the form $f_\omega(\omega) \alpha \omega^{-1}$ for all but the smallest masses. This is in approximate agreement with what is observed, and unfortunately this is a sign that the integral equation constituting the statistical model is poorly conditioned. This means that most "non-pathological" mass distributions will produce frequency distributions of approximately observed form, and that the process of elucidating mass distributions from frequency distributions is inherently inaccurate.

CONCLUSION

We estimate the mass function for the halo lenses by application of the iterative process with the observed frequency function, the integral kernel function, and an initial mass function estimate that is uniform on (0.01,3) solar masses. After eleven iterations the resulting mass function rises sharply toward smaller masses roughly as $f_M(M) \alpha M^{-1}$, before rolling over sharply near 0.1 solar masses. This structure appears to partially be due to the truncation of the frequency distribution at its high end. While the statistical model predicts that many of the lenses are of masses comparable to low-mass burning stars, there is little direct evidence for brown dwarfs or objects of lower mass.

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